

Acknowledgement of Country

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Any resources such as texts, websites and so on that may be referred to in this document are provided as examples of resources that teachers can use to support their learning programs. Their inclusion does not imply that they are mandatory or that they are the only resources relevant to the course.

Sample assessment task

Mathematics Methods – ATAR Year 11

Task 1 – Unit 1

Assessment type: Response

Content: Trigonometric functions (1.2.1 – 1.2.8), Functions and graphs (1.1.1 – 1.1.12)

Conditions:

Time for the task: Up to 60 minutes, in class, under test conditions

Note: while the Authority provides sample assessment tasks for guidance, it is the expectation of the Authority that teachers will develop tasks customised to reflect their school's context and the needs of the student cohort. This resource is available on a public website and use of the resource without modification may affect the integrity of the assessment.

Materials required:

Section One: Calculator-free. Standard writing equipment Section Two: Calculator-assumed. Calculator (to be provided by the student)

Other materials allowed:

Drawing templates, 1 A4 page of notes in Section Two

Marks available:	70 marks
Section One: Calculator-free	41 marks
Section Two: Calculator-assumed	29 marks

Task weighting:

6%

Section One: Calculator-free	(41 marks)
Suggested time: 35 minutes	
Question 1 Choose a suitable method to solve each of the equations below:	(7 marks)
(a) $3(2a-1)(4+a) = 0$	(2 marks)

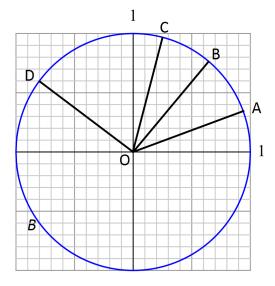
(b) 2t + 5 = 3(t - 7)

(c) $x^2 + 4x + 1 = 0$

(3 marks)

(2 marks)

The diagram below shows a unit circle, centre O. The rays OA, OB, OC and OD define the angles 20°, 50°, 75° and 143° as shown.



- (a) Use the unit circle to determine an approximate value for each of the following: (5 marks)
 - (i) sin 50°
 - (ii) cos 105°
 - (iii) sin 340°
 - (iv) tan 143°
- (b) Using the unit circle, explain why $\sin 50^\circ = \sin 130^\circ$ (3 marks)

(8 marks)

The function $y = 2x^2 + bx + c$ has a y intercept at (0, 2) and it has no roots. Determine the value of c and the range of possible values for b.

Use exact values to show that:

(a) $(\sin 45^\circ)^2 + 2\cos 30^\circ - \tan 60^\circ = \frac{1}{2}$

(b)
$$\tan\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{6}\right)\cos(\pi) = 0$$

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(3 marks)

(4 marks)

(2 marks)

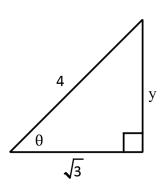
(2 marks)

(3 marks)

Question 5

Use the diagram to find the exact value of:

(a) *y*



(b) $\sin \theta$

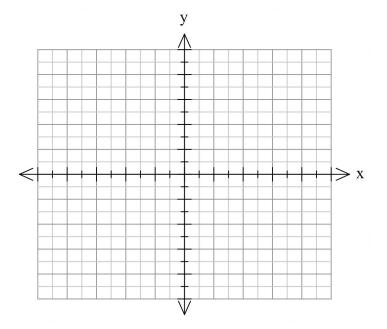
Ques	tion 6	(7 marks)
(a)	Determine the equation of the straight line that passes through the point (5, 3) and is	
	perpendicular to $x + 2y - 9 = 0$.	(3 marks)

(b) Find the value(s) of k if kx - 2y = 3 is parallel to 3x + (k + 7)y = -1. (4 marks)

Question 7 (5 marks) (a) Determine the coordinates of the root(s) and y intercept of the graph $y = x^2 + x - 6$. (2 marks)

(b) Sketch the graph on the axes below, clearly showing the coordinates of all significant points.

(3 marks)



Question 8

(4 marks)

Determine:

(a) the coordinates of the turning point of the graph of $y = 3(x + 4)^2 + 2$. (1 mark)

(b) the equation of the quadratic function that passes through the point (0, -9) and has a turning point at (2, 3). (3 marks)

Section Two: Calculator-assumed	(29 marks)
Suggested time: 25 minutes	
Question 9 Triangle <i>ABC</i> has sides $AB = 16.5$ cm, $BC = 12.8$ cm and $\angle CAB = 36^{\circ}$.	(5 marks)
(a) Determine the possible size(s) of $\angle ABC$	(3 marks)

(b) Hence determine the shortest possible length of the side *AC* (2 marks)

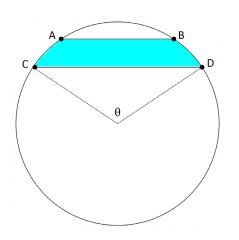
Question 10

(4 marks)

A triangle with an area of 36 m² has two equal sides with an included angle of 100°. Find the perimeter of the triangle.

(5 marks)

The length of the minor arc AB in the circle below is $\frac{16\pi}{3}$ cm and the circle has a radius of 12 cm. If the angle θ subtended by the arc CD is $\frac{3\pi}{5}$, determine the area of the shaded region.



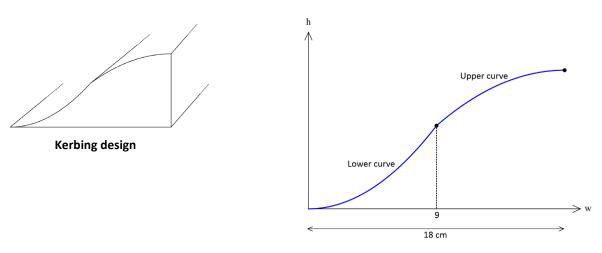
Question 12

(4 marks)

A line is inclined at an angle of 120° to the positive x-axis and cuts the y-axis at (0, -2). Determine the exact coordinates of the midpoint of the line segment determined by the x and y intercepts.

(5 marks)

Mark's Kerbing Company makes concrete kerbing. One design is shown below and its cross-section is modelled on the axes. The curve is created by two quadratic functions that intersect when w = 9. The upper curve is modelled by the equation $h = \frac{-2w^2}{27} + \frac{8w}{3} - 9$, where w and h are measured in centimetres.



The equation of the lower curve is of the form $h = aw^2$.

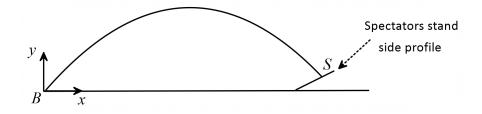
(a) Determine the coordinates of the point where the upper and lower curves intersect. (1 mark)

(b) Determine the equation of the lower curve.

(2 marks)

(c) The volume of concrete used is directly proportional to the length of kerbing made.
 A length of 2 m requires 0.036 m³ of concrete. Determine the volume of concrete needed to make a 1.2 km length of kerbing.
 (2 marks)

In a cricket match, a batsman at B hits a ball toward the spectators stand.



Taking B as the origin of the coordinate axes, the path of the ball can be modelled by the equation y = 0.002x(115 - x).

The side profile of the spectator stand is given by the equation y = 0.1x - 10 for $100 \le x \le 115$, where x and y are measured in metres.

(a) Find the maximum height reached by the cricket ball. (1 mark)

(b) A fielder catches the ball 6 metres from the batsman. At what height was the ball? (2 marks)

(c) The ball clips the top of a 3.8 m high sight screen near the spectators stand at the edge of the field. Find the distance the sight screen is from the batsman.
 (2 marks)

(d) The cricket ball lands in the spectators stand at S. Determine the coordinates of S. (1 mark)

Marking key for sample assessment task 1 – Unit

Section One: Calculator-free

Question 1

Solution	Marks
(a) $3(2a-1)(4+a) = 0$	
2a - 1 = 0 or $4 + a = 0$	
$a = \frac{1}{2}$ or $a = -4$	
Identifies that a solution can be found from each factor	1
Determines the two correct solutions for <i>a</i>	1
(b) $2t + 5 = 3(t - 7)$	
2t + 5 - 3t + 21 = 0	
-t + 26 = 0	
t = 26	
Expands brackets correctly	1
Solves for <i>t</i>	1
(c) $x^2 + 4x + 1 = 0$	
$(x+2)^2 - 4 + 1 = 0$	
$(x+2)^2 - 3 = 0$	
$x + 2 = \pm\sqrt{3}$	
$x = \pm \sqrt{3} - 2$	
Completes the square (or substitutes values into the quadratic formula) correctly	1
Solves for <i>x</i>	1
Gives two solutions for x	1
Subtotal	/7

Solution	Marks
(a) (i) sin 50° ≈ 0.76	
(ii) cos 105° ≈ - 0.24	
(iii) sin 340° ≈ – 0.33	
(iv) $\tan 143^\circ = \frac{\sin 143^\circ}{\cos 143^\circ} \approx \frac{0.6}{-0.8} \approx -0.75$	
Reads an approximate value from the unit circle for sine and cosine (1 each)	3
Expresses tangent as a function of sine and cosine	1
Uses values from the unit circle to determine tangent	1
(b) Sine of an angle is the y coordinate of the point of intersection of the terminal ray of the	e angle and the
unit circle. The ray that forms the angle of 130° is a reflection about the vertical axis of	the ray OB,
representing 50° and therefore has the same y coordinate.	
Recognises that 130° is a reflection about the vertical axis of the ray representing 50°	1
States that sine of an angle is the y coordinate of the point of intersection	1
Identifies that both angles have the same y coordinate at the point of intersection with the	1
unit circle	
Subtotal	/8

Solution		Marks	
<i>c</i> = 2			
$b^2 - 4ac < 0$			
$b^2 - 16 < 0$			
$b^2 < 16$			
-4 < b < 4			
Identifies the y intercept as the value for c		1	
Recognises that the discriminant must be less than 0		1	
Correctly identifies the range of values for b		1	
	Subtotal		/3

Solution	Marks
(a) $(\sin 45^\circ)^2 + 2\cos 30^\circ - \tan 60^\circ$	
$= \left(\frac{1}{\sqrt{2}}\right)^2 + 2\left(\frac{\sqrt{3}}{2}\right) - \sqrt{3}$	
$=\frac{1}{2}+\sqrt{3}-\sqrt{3}$	
$=\frac{1}{2}$	
Determines exact values of sin θ , cos θ and tan θ correctly	1
Justifies the solution numerically	1
(b) $\tan\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{6}\right)\cos(\pi)$	
$= \left(\frac{1}{\sqrt{3}}\right) \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right) (-1)$	
$=\frac{1}{2}-\frac{1}{2}$	
= 0	
Determines exact values of sin θ , cos θ and tan θ correctly	1
Justifies the solution numerically	1
Subtotal	/4

Solution	Marks
(a) $y^2 = 4^2 - \sqrt{3}^2$	
$y^2 = 13$	
$y = \sqrt{13}$	
Determines the relationship between the sides of the right-angle triangle	1
Correctly determines the length of the side y	1
(b) $\sin\theta = \frac{\sqrt{13}}{4}$	
Expresses sin θ as a correct ratio of the sides	1
Subtotal	/3

Solution	Marks
(a) $x + 2y - 9 = 0$	·
$y = -\frac{1}{2}x + \frac{9}{2}$	
$m_1 = -\frac{1}{2}$ therefore new line has $m_2 = 2$ and equation $y = -2x + c$	
Given (5, 3)	
3 = 2(5) + c	
c = -7	
Equation of line is $y = 2x - 7$	
Determines gradient of the given line correctly	1
Determines gradient of the required line as the opposite, reciprocal of the original line	1
Determines the y intercept and equation of the required line	1
(b) Line 1: $kx - 2y = 3$	
$y = -\frac{k}{2}x - \frac{3}{2}$	
Line 2: $3x + (k + 7)y = -1$	
$y = \frac{-3x}{(k+7)} - \frac{1}{(k+7)}$	
If parallel then $\frac{k}{2} = \frac{-3}{k+7}$	
k(k+7) = -6	
$k^2 + 7k + 6 = 0$	
(k+1)(k+6) = 0	
k = -1 or $k = -6$	
Determines an expression for the gradient of Line 1 and Line 2	1
Identifies that the expressions for the gradient are equal	1
Determines a quadratic equation to solve	1
Solves for k	1
Subtotal	/7

Solution	Marks
(a) At $x = 0$, $y = -6$, y intercept at $(0, -6)$	
At $y = 0$, $x^2 + x - 6 = 0$	
(x+3)(x-2) = 0	
x = -3 or x = 2	
Roots at (-3,0) and (2,0)	
Determines the coordinates of the <i>y</i> -intercept	1
Determines both roots	1
Turning point at $x = \frac{-3+2}{2} = -\frac{1}{2}$ $y = \left(-\frac{1}{2}\right)^2 + \frac{1}{2} - 6$ $= -6\frac{1}{4}$ $\left(-\frac{1}{2}, -6\frac{1}{4}\right)$ $\left(-\frac{1}{2}, -6\frac{1}{4}\right)$	
Determines the coordinates of the turning point	1
Draws a smooth parabolic curve	1
Clearly shows/labels all critical points	1
Subtotal	/5

Solution	Marks	
(a) (-4,2)		
Identifies coordinates of the turning point correctly from the equation	1	
(b) $y = a(x-2)^2 + 3$		
When $x = 0$, $y = 4a + 3$		
-9 = 4a + 3		
a = -3		
$y = -3(x-2)^2 + 3$		
Expresses the equation in turning point form using (2, 3) correctly	1	
Determines a value for a	1	
Writes the equation of the quadratic function correctly	1	
Subtotal		/4
Section One Total		/41

Section Two: Calculator-assumed

Question 9

Sc	plution	Marks
(a) $A \xrightarrow{16.5 \text{ cm}} B$ F_{20} $F_$	$\frac{\sin C}{16.5} = \frac{\sin 36^{\circ}}{12.8}$ $\sin C = \frac{16.5 \sin 36^{\circ}}{12.8}$ $C = 49.26^{\circ} \text{ or } 130.74^{\circ} (180^{\circ} - 49.26^{\circ})$ $\angle ABC = 94.74^{\circ} \text{ or } 13.26^{\circ}$	
Determines one possibility for the size of ar	gle B or one possibility for the size of angle C	1
Determines an alternative value for the size	of angle C	1
Identifies two possible values for the size of	angle ABC	1
(c) Shortest length when $\angle ABC = 13.26^{\circ}$ $AC = \sqrt{16.5}$ $AC = \sqrt{24.9}$	$5^{2} + 12.8^{2} - 2(16.5)(12.8)\cos 13.26^{\circ}$	
AC = 5.0 cm		
Recognises that the smallest angle will define	ne the shortest side	1
Determines the length of side AC		1
	Subtotal	

	Solution	Marks
a loos b	$36 = \frac{1}{2}a^{2} \sin 100^{\circ}$ $a^{2} = 73.11$ $a = 8.55 \text{ m}$ $b = \sqrt{2(8.55)^{2} - 2(8.55)^{2} \cos 100^{\circ}} \text{ or } \frac{b}{\sin 100^{\circ}} = \frac{8}{\sin 100^{\circ}}$ $b = 13.10 \text{ m}$ Perimeter = 2(8.55) + 13.10 = 30.20 \text{ m}	3.55 n 40°
Writes a correct expression relating	ng the area and unknown sides	1
Determines the length of the equa	al sides	1
Determines the length of the third	l side	1
Determines the perimeter of the t	riangle	1
	Subtotal	/4

Solution		Marks
Arc length(AB) = 12β	Area(segment CD) = $\frac{1}{2}(12)^2 \left(\frac{3\pi}{5} - sin\right)$	$\left(\frac{3\pi}{5}\right)$
$\frac{16\pi}{3} = 12\beta$	= 67.24 cm	
$\beta = \frac{4\pi}{9}$	Shaded area $= 67.24 - 29.62$	
Area(segment AB) = $\frac{1}{2}(12)^2\left(\frac{4\pi}{9} - \sin\frac{4\pi}{9}\right)$	$= 37.62 \text{ cm}^2$	
= 29.62 cm		
Determines the size of the minor angle subtended b	by the arc AB	1
Determines the area of the minor segment AB		1
Determines the area of the minor segment CD (or the sector CD)		1
Subtracts the segment areas (CD – AB) to find the shaded region		1
or alternate method (sector CD – segment AB – \triangle C	CDO)	Ţ
Determines the correct area of the shaded region (including units)		1
	Subtotal	/5

Solution		Marks
$\xrightarrow{120^{\circ}} x x-i$	uation of line: $y = mx - 2$ $m = \tan 60^{\circ}$ $= \sqrt{3}$ $y = \sqrt{3}x - 2$ htercept at $y = 0$: $0 = \sqrt{3}x - 2$ $x = \frac{2}{\sqrt{3}}$ dpoint of $(0, -2)$ and $(\frac{2}{\sqrt{3}}, 0)$ is $(\frac{1}{\sqrt{3}}, -1)$	
Determines gradient of line correctly		1
Determines coordinates of the <i>x</i> -intercept		1
Determines coordinates of midpoint		1
Expresses midpoint correctly as a coordinate using e	exact values	1
	Subtotal	/4

Solution		Marks	
(a) (9,9)	I		
Expresses point of intersection of curves correctly as a coordinate		1	
(b) $h = aw^2$			
9 = 81a			
$a = \frac{1}{9}$			
$h = \frac{w^2}{9}$			
Determines the value of a		1	
Expresses the equation correctly using a		1	
(c) $V \propto l$			
V = kl			
0.036 = 2k			
k = 0.018l			
$V = 0.018 \times 1200$			
V = 21.6			
21.6 m ³ of concrete required			
Determines the constant of proportionality		1	
Determines the volume of concrete correctly using correct units of volume		1	
Su	ubtotal		/5

Solution	Marks
(a) Using CAS	
Max is 6.6125 m	
Determines height correctly	1
(b) At $x = 6$, $y = 1.308$	
Ball was 1.308 m high	
Determines <i>y</i> -coordinate when $x = 6$	1
Expresses the solution correctly including units	1
(c) At $y = 3.8$, $x = 20$ or $x = 95$	·
Fielder is 95 m from the batsman	
Determines both <i>x</i> -coordinates when $y = 3.8$	1
Expresses the correct solution including units	1
(d) (110.32, 1.032)	·
Determines point of intersection of the line and the parabola	1
Subtotal	/6
Section Two Total	/29
Final Total	/70

Sample assessment task

Mathematics Methods – ATAR Year 11

Task 2 – Unit 1

Assessment type: Investigation

Conditions:

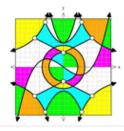
The investigation requires the use of the mathematical thinking process. The task will be completed using two sessions in class under supervised conditions. Students will then have one week to complete their investigation at home. Students may use any appropriate technology.

Task weighting:

7% of the school mark for this pair of units

Mathematical Art

You have been employed to design an art piece that highlights at least two mathematical relations and best showcases your knowledge of the functions and relations studied in Unit 1.1.



Using your knowledge of functions together with graphing software, such as

Desmos <u>https://www.desmos.com/</u>, design your own art piece. Some examples of mathematical art can be found at <u>https://www.desmos.com/art</u>.

Your investigation should make use of the mathematical thinking process:

- interpreting the task and the key information
- choosing the mathematics which could help to complete the task
- applying existing mathematical knowledge and strategies to obtain a solution
- interpreting the results in relation to the context
- communicating the solution to the problem as required.

You will then write a report to showcase your design. As you write your report, take care to clearly identify the underlying mathematics used throughout the process.

Your report should include the following:

- an introduction, that clearly defines the purpose of the task, identifies key information, any assumptions made and an outline of your strategy (6 marks)
- evidence of the application of mathematical knowledge and strategies, including calculations and results using appropriate representations (graphs, tables, formulae etc.)
 (21 marks)
- your final design communicated in a systematic and concise manner, including analysis and interpretation in the context of the problem and consideration of the reasonableness and limitations of the results.
 (12 marks)
- use of correct mathematical conventions, symbols and terminology. (5 marks)

The format of the report may be written or digital.

Marking key for sample assessment task 2 – Unit 1

Description	Marks available	е
Introduction		
Succinctly writes a general introduction that summarises the aim of the investigation	2	
Identifies that a variety of relations will need to be used to define the art piece	1	
Recognises the need to restrict the domains of the relations using points of intersection	1	
Identifies that special features of each graph needs to be considered in the context of the art piece	1	
Describes or sketches a proposed design	1	
Subtotal		/6
Application of mathematical knowledge and strategies		
Locates the art piece on the Cartesian plane	1	
Chooses at least two different functions/relations for the design	2	
Defines the equation to describe at least different two functions/relations	2	
Defines the domain for at least two different functions/relations	2	
Chooses a third function/relation for the design, defines the equation and a suitable domain	3	
Chooses a fourth function/relation for the design, defines the equation and a suitable domain	3	
Chooses further functions/relations for the design, defines the equations and suitable domains	3	
Includes a range of functions/relations (two types – 1 mark, three types – 2 marks, more than three types – 3 marks)	3	
Provides mathematical evidence to support the development of the art piece (sometimes – 1 mark, consistently – 2 marks)	2	
Subtotal	/2	21
Analysis and interpretation		
Draws and labels at least two functions/relations clearly showing any points of intersection and the relationship to the domain and range	3	
Draws and labels a third function/relation clearly showing any points of intersection and the relationship to the domain and range	3	
Draws and labels a fourth function/relation clearly showing any points of intersection and the relationship to the domain	3	
Draws and labels further functions/relations clearly showing points of intersection and the relationship to the domain	3	
Subtotal	/1	12
Use of correct mathematical conventions, symbols and terminology		
Graphs are correctly labelled and displayed appropriately (sometimes – 1 mark, consistently – 2 marks)	2	
Uses mathematical language throughout the investigation (sometimes – 1 mark, consistently – 2 marks)	2	
Presents investigation in a systematic and concise way	1	
Subtotal	/	/5
	-	44